

Bouncer Project

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Contents

1	Abstract	3
2	Introduction:	4
3	Method:	5
3.1	Coefficient of restitution	5
3.1.1	Calculating the coefficient of restitution	5
3.1.2	Calculating the uncertainty in the coefficient of restitution	6
3.2	Initial velocity at the end of the tube	7
3.3	Distance to the first bounce (d_2)	8
3.3.1	Initial velocity (At the exit of the PVC pipe)	9
3.3.2	Looking at the vertical motion:	9
3.3.3	Using the t_1 value calculated to look at horizontal motion	10
3.4	Distance between first and second bounce (d_3)	11
3.4.1	Calculating the velocity of the ball after the collision:	11
3.4.2	Using the velocities calculated to work out the distance (d_3)	12
3.5	Total predicted distance	13
4	Results:	13
4.1	Absolute Uncertainties	13
4.2	Determining the uncertainty in d_t	14
4.2.1	Coefficient of restitution:	14
4.2.2	Angle of incline of PVC pipe ($^\circ$)	14
4.2.3	Length of the PVC pipe (l)(m)	15
4.2.4	Height of the PVC pipe from the ground (h_2)(m)	15
4.2.5	Sensitivity analysis conclusion	16
5	Method of experiment:	16
5.1	Photo of target sheets	16
5.2	Experimental method	17

5.3	Assumptions	18
5.3.1	Energy lost in the PVC pipe	18
5.3.2	Initial Kinetic Energy	20
5.3.3	Friction during the bounce	21
5.3.4	Revised additions to sensitivity analysis	22
5.3.5	Further assumptions in calculations and experiment	23
6	Conclusion	23

1 Abstract

The report predicts the horizontal distance travelled (d_t) by a steel ball after rolling through a PVC pipe, and bouncing on a carpeted surface, and then hitting the carpet again. Using energy balance, kinematics and projectile motion, the theoretical distance of 1.773m was obtained. This was then tested and an experimental value of 1.687m was found. A sensitivity analysis then found the coefficient of restitution was the largest contributor to the uncertainty in d_t which in total was $\pm 0.063\text{m}$. Further effects such as energy lost to sound, an accidental initial kinetic energy and friction during the bounce were analysed, giving a new theoretical distance of approximately 1.81m. When the results were compared, it suggested that the assumptions made inside the pipe and during the collision caused the variation.

2 Introduction:

This report lays out the experiment, calculations, data, and observations used as part of the bouncer challenge. The challenge was to calculate the distance traveled by a steel ball, which first rolled along a tilted PVC pipe, before bouncing once and making contact with the ground a second time. The setup is as shown below in Figure 1.¹

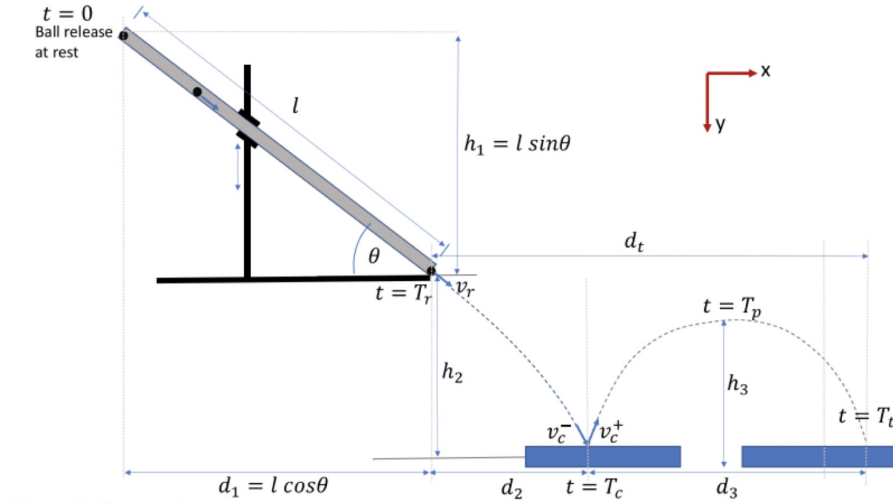


Figure 1: Game setup

This experiment also included the calculation of the coefficient of restitution, and documents how this was done.

Applying the principles of energy conservation and kinematics, the horizontal distance (d_t) can be theorized before comparing it with the experimental data collected.

All equations in this report were cited from (Nanayakkara, T. and Ghajarim M (2019) MECHANICS lecture notes. Dyson School of Design Engineering, Imperial College London

The assumptions made in this experiment are that the ball rolls without slipping in the PVC pipe (i.e. only rolls) and that there is no friction in the PVC pipe. It is assumed that the acceleration due to gravity is constant (9.81 m s^{-2}) and that there is no horizontal acceleration due to air resistance. We also assume that the ball is released from rest, therefore the initial kinetic energy is 0. Finally, it is assumed that the steel ball's collision and that the coefficient of restitution only applies to the vertical component of velocity, and that the steel ball is a perfect sphere.

Angle of tube, θ	=	30°
Length of tube, L	=	1.5 m
Height of lower end, h_2	=	0.695 m
Coefficient of restitution, e	=	0.469 ± 0.027 (calculated in Method)
Mass of ball, m	=	$(4.19 \pm 0.0027) \text{ g}$
Diameter of ball, d	=	0.01 m
Acceleration due to gravity, g	=	9.81 m s^{-2}

¹Nanayakkara, T. (2025) Bouncer Challenge Description. Dyson School of Design Engineering, Imperial College London

3 Method:

3.1 Coefficient of restitution

3.1.1 Calculating the coefficient of restitution

To calculate the coefficient of restitution, I conducted a drop test with the steel ball. The ball was released from a height of 1 m, measured with a metre rule, and a slow motion camera, perpendicular to said rule. The setup used is shown in Figure 2.

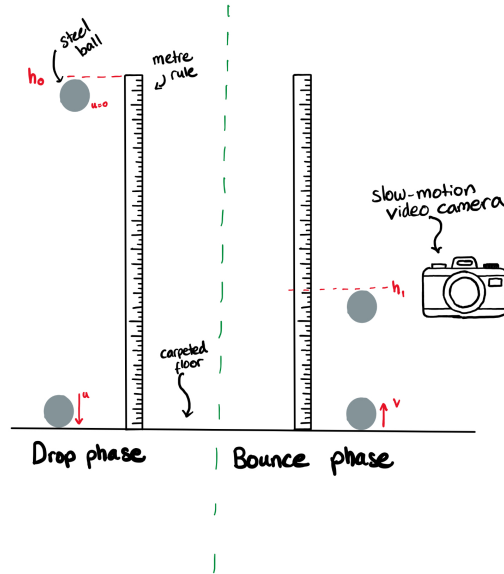


Figure 2: Experimental setup for calculating the coefficient of restitution.

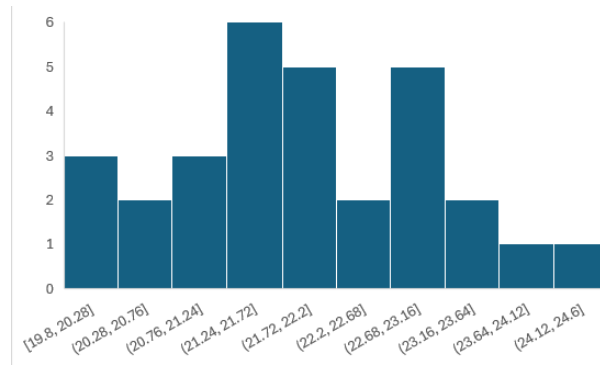


Figure 3: Histogram of drop heights

Once these results were gathered, I calculated the average bounce height using data from Figure 3:

$$H_{1_{avg}} = 0.22m$$

Therefore, using the average bounce height and initial drop height, I can calculate e :

$$e = \sqrt{\frac{H_{1avg}}{H_0}} \Rightarrow e = \sqrt{\frac{22}{100}}$$

$$e = 0.469$$

3.1.2 Calculating the uncertainty in the coefficient of restitution

Absolute uncertainty in H_1 :

$$\text{Absolute unc. } H_1 = \frac{\text{Range}_{H_1}}{2}$$

$$\text{Absolute unc. } H_1 = \frac{24.6 - 19.8}{2} = 2.4\text{cm}$$

Percentage uncertainty in H_1 :

$$H_1 = 0.22\text{m}$$

$$H_1 = (0.22 \pm 2.4)\text{cm}$$

$$\therefore \text{Percentage unc. } H_1 = \frac{2.4}{0.22} \cdot 100 = 10.91\%$$

On the other hand, uncertainty can come from the measurements using the metre rule. This is a fixed value which is $\pm 0.5\text{mm}$.

Calculating the uncertainty from the metre rule:

$$\text{Absolute unc.} = \frac{\text{smallest resolution}}{2}$$

$$\text{Absolute unc.} = \frac{1}{2} = 0.5\text{mm}$$

$$\therefore \text{Percentage unc.} = \frac{0.5}{100} \cdot 100 = 0.5\%$$

Adding the two percentage uncertainties together and then halving, as $e = \sqrt{\frac{H_1}{H_0}}$:

$$(10.91\% + 0.5\%) \cdot \frac{1}{2}$$

$$\therefore e = 0.469 \pm 5.7\%$$

$$\text{where } e = 0.469 \pm 0.027$$

3.2 Initial velocity at the end of the tube

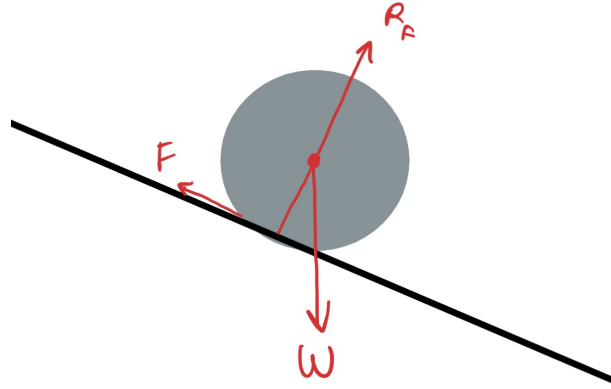


Figure 4: The free body diagram of the steel ball as it rolls through the tube.

As seen above, Figure 4 shows the forces acting on the ball as it rolls down the tube, which is key to the following calculations.

To calculate the initial speed of the ball, when the ball leaves the pipe, we must apply the conservation of energy, assuming that energy is not lost to any other form such as heat or sound, or that the ball doesn't slip. For this calculation, we are assuming the ball is released from rest.

If energy was lost during the ball traveling through the pipe, the velocity would be lower than the theoretical value, meaning the final distance will be shorter overall.

The parameters that I will be using are listed in the table below.

Angle of tube, θ	=	30°
Length of tube, L	=	1.5 m
Height of lower end, h_2	=	0.695 m
Coefficient of restitution, e	=	0.469 ± 0.027 (calculated in Method)
Mass of ball, m	=	(4.19 ± 0.0027) g
Diameter of ball, d	=	0.01 m
Acceleration due to gravity, g	=	9.81 m s^{-2}
Vertical height of tube, h_1	=	$1.5 \sin 30$
Horizontal height of tube, d_1	=	$1.5 \cos 45$

GPE	=	Gravitational potential energy
KE_t	=	Kinetic energy (Translational)
KE_r	=	Kinetic energy (Rotational)

Using the conservation of energy:

The energy put into the system will be equal to the energy in the system when the ball leaves the PVC pipe.

$$Before_{PE} = After_{PE}$$

$$\therefore GPE_1 + KE_{t_1} + KE_{r_1} = GPE_2 + KE_{t_2} + KE_{r_2}$$

By simplifying this into changes in energy, we can sum this as follows:

$$\Delta GPE = \Delta KE_t + \Delta KE_r$$

Using the equations for GPE , KE_r , KE_t , we can sub in m , g , h_1 , and d to form the following equation sets.

$$mgh_1 = \frac{1}{2}mv_r^2 + \frac{1}{2}I\omega^2,$$

Assuming that the ball is a perfect sphere allows us to use the formula $I = \frac{2}{5}mr^2$ and $\omega = \frac{v_r}{r}$, where (I = moment of inertia), (ω = angular velocity)

$$mgh_1 = \frac{1}{2}mv_r^2 + \frac{1}{2} \cdot \frac{2}{5}(mr^2)\left(\frac{v_r}{r}\right)^2,$$

Simplifying this equation:

$$gh_1 = \frac{1}{2}v_r^2 + \frac{1}{5}v_r^2 = \frac{7}{10}v_r^2$$

$$\frac{10}{7}gh_1 = v_r^2 \quad \Rightarrow \quad v_r = \sqrt{\frac{10}{7}gh_1}.$$

Using h_1 from the parameters listed above, the solution for v_r can be found:

$$v_r = \sqrt{\frac{10}{7}(9.81)(1.5 \sin 30)} = 3.242 \text{ m s}^{-1}.$$

3.3 Distance to the first bounce (d_2)

For this subsection, I have calculated the distance traveled from the PVC pipe exit, to the first bounce (d_2). Figure 4 shows the exit velocities of the ball as components.

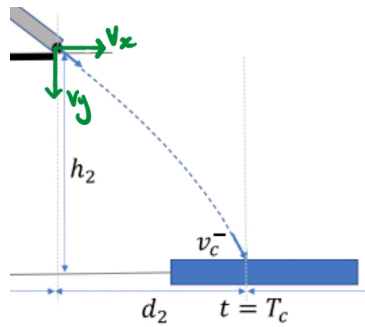


Figure 5: Velocity acting on the ball as it exits the PVC pipe

To calculate this, kinematics must be used; the equations and steps I followed are laid out below. When the ball leaves the pipe, all horizontal forces stop, and we assume there is no air resistance, therefore the horizontal acceleration is 0.

3.3.1 Initial velocity (At the exit of the PVC pipe)

$$u_x = v_r \cos \theta = 3.242 \cos 30^\circ = 2.808 \text{ m s}^{-1}$$

$$u_y = v_r \sin \theta = 3.242 \sin 30^\circ = 1.621 \text{ m s}^{-1}$$

3.3.2 Looking at the vertical motion:

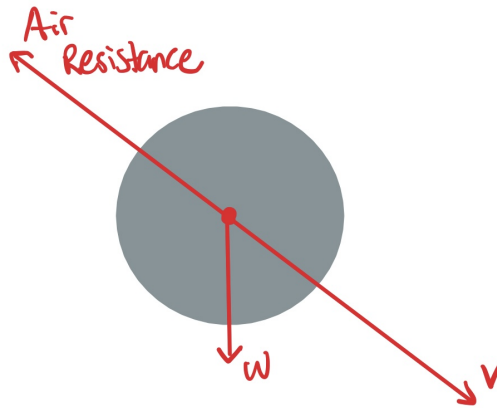


Figure 6: The free body diagram of the steel ball as it falls

For the vertical motion kinematics, the parameters I used are in the table below:

Initial vertical velocity, u_y	=	1.621 m s^{-1}
Vertical acceleration, a_y	=	9.81 m s^{-2}
Final vertical velocity, v_y	=	?
Time taken till first bounce, t_1	=	?
Vertical displacement, s_y	=	0.695 m

Using a formula to find the time taken for the ball to hit the ground, using the three variables we know, and leaving time as the required variable, we can follow these steps:

First, the equation I will use is:

$$s = u_y t_1 + \frac{1}{2} a_y t_1^2,$$

Using this equation, we can sub our values in, forming the equation below:

$$0.695 = 1.621 t_1 + 4.905 t_1^2$$

Rearranging to put the equation in a more understandable form:

$$4.905 t_1^2 + 1.621 t_1 - 0.695 = 0.$$

Solving the equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_1 = \frac{-1.621 \pm \sqrt{(1.621)^2 - 4(4.905)(-0.695)}}{2 \times 4.905}$$

Which gives the answers: $t_1 = 0.246 \text{ s}$ and $t_1 = -0.576 \text{ s}$ The negative solution can be ignored as it is not possible.

$\therefore$ the solution for t_1 is: $t_1 = 0.246 \text{ s}$

Setting this value of t_1 aside, a new equation can be used to calculate the final vertical velocity which will be needed later:

$$v_y^2 = u_y^2 + 2as_y$$

Subbing in the values from the table above and simplifying will produce the answer for v_y :

$$v_y^2 = 1.621^2 + 2 \cdot (9.81) \cdot (0.695)$$

$$v_y = 4.033 \text{ m s}^{-1}$$

3.3.3 Using the t_1 value calculated to look at horizontal motion

For the horizontal motion kinematics, the parameters I used are in the table below:

Initial horizontal velocity, u_x	=	2.808 m s^{-1}
Horizontal acceleration, a_x	=	0 m s^{-2}
Final horizontal velocity, v_x	=	2.808 m s^{-1}
Time taken till first bounce, t_1	=	0.246 s
Horizontal displacement, d_2	=	?

Using the time I calculated above, we can produce a formula to calculate the horizontal distance traveled by the steel ball until the first bounce:

$$d_2 = u_x \cdot t_1 + \frac{1}{2} \cdot a_x \cdot t_1^2$$

We are assuming that there is no air resistance acting on the ball, therefore no horizontal forces, which means there is no horizontal acceleration, allowing us to simplify the equation:

$$d_2 = u_x \cdot t_1$$

Substituting in the values from the table above, we can

$$d_2 = 2.808 \cdot 0.246 = 0.690 \text{ m.}$$

3.4 Distance between first and second bounce (d_3)

In this section, I am using the coefficient of restitution I calculated, as well as the velocities just before the bounce which I calculated in the previous section, to find the velocities after the collision, and then apply these to new kinematic equations, to calculate d_3 . Below is a figure laying out the first bounce.

Following the assumptions in this section, it is assumed there is no friction acting on the ball's horizontal velocity component, and that the coefficient of restitution only acts vertically, the actual d_t value will be bigger than the theoretical value calculated, due to the fact that the collision will cause the ball to spin, increasing horizontal velocity due to rotational kinetic energy being converted upon impact with the carpet floor.

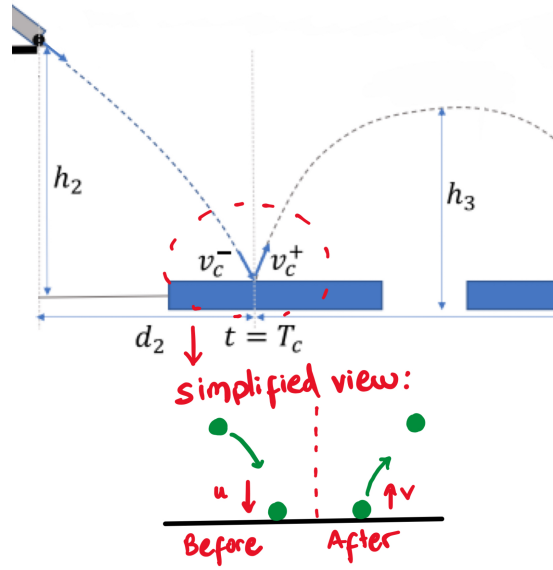


Figure 7: The change in motion of the ball due to the collision with the carpet floor.

3.4.1 Calculating the velocity of the ball after the collision:

The final velocity of the steel ball before the collision can be separated into two components:

$$u_x = 2.808 \text{ m s}^{-1}$$

$$u_y = 4.033 \text{ m s}^{-1}$$

Assuming there is no friction during the contact between the ball and carpet floor, as well as the coefficient of restitution only impacting the vertical component of velocity ($\therefore u_x$ remains the same), we can calculate the velocities after the collision (where v_y acts in the opposite direction to u_y):

$$v_x = 2.808 \text{ m s}^{-1}$$

$$v_y = e \cdot u_y$$

Now substituting in our value of u_y and e from our earlier calculation:

$$\therefore v_y = 0.469 \cdot 4.033 = 1.891 \text{ m s}^{-1}$$

This is displayed in the figure below, showing the change in initial velocities to final velocities:

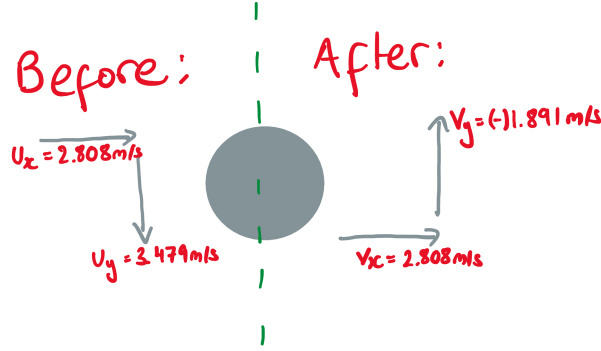


Figure 8: The change in velocity (components) of the steel ball, before and after the collision with the carpet floor.

3.4.2 Using the velocities calculated to work out the distance (d_3)

For the **vertical motion** kinematics, the parameters I used are in the table below:

Initial vertical velocity, u_y	=	1.891 m s^{-1}
Vertical acceleration, a_y	=	-9.81 m s^{-2}
Time taken till second bounce, t_2	=	?
Vertical displacement, s_y	=	0 m

Using a formula to find the time taken for the ball to hit the ground a second time, using the three variables we know, and leaving time as the required variable, we can follow these steps:

First, the equation I will use is:

$$s = u_y t_2 + \frac{1}{2} a_y t_2^2,$$

Using this equation, we can sub our values in, forming the equation below:

$$0 = 1.891 t_2 + \frac{1}{2} \cdot (-9.81) t_2^2$$

This simplifies to:

$$4.905 t_2^2 - 1.891 t_2 = 0.$$

Solving the equation:

$$\begin{aligned} t_2(4.905 t_2 - 1.891) &= 0 \\ \therefore t_2 &= 0 \text{ s}, 0.386 \text{ s} \end{aligned}$$

The first solution can be ignored as it is the starting value. (i.e. instantaneously after the first collision)

$\therefore$ the solution for t_2 is: $t_2 = 0.386 \text{ s}$

For the **horizontal motion** kinematics, the parameters I used are in the table below (including the value of t_2 I just calculated:

Initial horizontal velocity, u_x	=	2.808 m s^{-1}
Horizontal acceleration, a_x	=	0 m s^{-2}
Final horizontal velocity, v_x	=	2.808 m s^{-1}
Time taken till second bounce, t_2	=	0.386 s
Horizontal displacement, d_3	=	?

Using the time I calculated above, we can produce a formula to calculate the horizontal distance traveled by the steel ball from the first bounce until the second:

$$d_3 = u_x \cdot t_2 + \frac{1}{2} \cdot a_x \cdot t_2^2$$

Keeping the same assumptions as before regarding horizontal acceleration, we can simplify the equation:

$$d_3 = u_x \cdot t_2$$

Substituting in the values from the table above, we can:

$$d_3 = 2.808 \cdot 0.386 = 1.083 \text{ m.}$$

3.5 Total predicted distance

$$d_t = d_2 + d_3 = 0.690 + 1.083 = 1.773 \text{ m. (3 d.p.)}$$

4 Results:

4.1 Absolute Uncertainties

All the uncertainties in the d_t calculation come from the initial setup, as well as the measurement of e . The absolute uncertainties are listed below in the table:

Value	Result	Uncertainty	Upper	Lower
Coefficient of Restitution	0.469	± 0.027	0.496	0.442
Angle of incline of PVC pipe ($^\circ$):	30°	$\pm 0.005^\circ$	30.005°	29.995°
Length of the PVC pipe (m)	1.5 m	$\pm 0.0005 \text{ m}$	1.5005 m	1.4995 m
Height of the PVC pipe from the ground (m)	0.695 m	$\pm 0.0005 \text{ m}$	0.6955 m	0.6945 m

Table 1: A table laying out the uncertainty in d_t

4.2 Determining the uncertainty in d_t

To calculate this total value of d_t , a sensitivity analysis must be carried out. By calculating the percentage uncertainty impact of each value on d_t , we can sum them all and therefore figure out the range in d_t . We can do this by using the bounds and subbing them into my calculations to produce a range of values for d_t .

4.2.1 Coefficient of restitution:

Using the method laid out, the upper and lower values of d_t due to e are:

$$\begin{aligned}\text{Upper bound } (d_t) &= 1.835 \text{ m} \\ \text{Lower bound } (d_t) &= 1.711 \text{ m}\end{aligned}$$

For these values, we can calculate an absolute uncertainty due to the coefficient of restitution:

$$\text{Absolute unc. } d_t = \frac{1.835 - 1.711}{2} = 0.062 \text{ m}$$

$$\therefore d_t = (1.773 \pm 0.062) \text{ m}$$

To calculate the percentage uncertainty, the following steps are taken:

$$\frac{\text{Absolute unc. } d_t}{d_t} \cdot 100 = \text{Percentage unc. } d_t$$

$$\therefore \text{Percentage unc. } d_t = \frac{0.062}{1.773} \cdot 100 = 3.52\%$$

4.2.2 Angle of incline of PVC pipe ($^\circ$)

Using the method laid out, the upper and lower values of d_t due to the angle of the pipe are:

$$\begin{aligned}\text{Upper bound } (d_t) &= 1.77294 \text{ m} \\ \text{Lower bound } (d_t) &= 1.77290 \text{ m}\end{aligned}$$

For these values, we can calculate an absolute uncertainty due to the angle:

$$\text{Absolute unc. } d_t = \frac{1.77294 - 1.77290}{2} = 0.00002$$

$$\therefore d_t = (1.773 \pm 0.00002) \text{ m}$$

To calculate the percentage uncertainty, the following steps are taken:

$$\frac{\text{Absolute unc. } d_t}{d_t} \cdot 100 = \text{Percentage unc. } d_t$$

$$\therefore \text{Percentage unc. } d_t = \frac{0.00002}{1.773} \cdot 100 = 0.0012\%$$

4.2.3 Length of the PVC pipe (l)(m)

Using the method laid out, the upper and lower values of d_t due to the length of the pipe are:

$$\begin{aligned} \text{Upper bound } (d_t) &= 1.773\,20\text{ m} \\ \text{Lower bound } (d_t) &= 1.772\,64\text{ m} \end{aligned}$$

For these values, we can calculate an absolute uncertainty due to the length:

$$\text{Absolute unc. } d_t = \frac{1.77320 - 1.77264}{2} = 0.000278\text{m}$$

$$\therefore d_t = (1.773 \pm 0.000278)\text{m}$$

To calculate the percentage uncertainty, the following steps are taken:

$$\frac{\text{Absolute unc. } d_t}{d_t} \cdot 100 = \text{Percentage unc. } d_t$$

$$\therefore \text{Percentage unc. } d_t = \frac{0.000278}{1.773} \cdot 100 = 0.0157\%$$

4.2.4 Height of the PVC pipe from the ground (h_2)(m)

Using the method laid out, the upper and lower values of d_t due to the height of the pipe are:

$$\begin{aligned} \text{Upper bound } (d_t) &= 1.7722\text{ m} \\ \text{Lower bound } (d_t) &= 1.7736\text{ m} \end{aligned}$$

For these values, we can calculate an absolute uncertainty due to h_2 :

$$\text{Absolute unc. } d_t = \frac{1.7736 - 1.7722}{2} = 0.00067\text{m}$$

$$\therefore d_t = (1.773 \pm 0.00067)\text{m}$$

To calculate the percentage uncertainty, the following steps are taken:

$$\frac{\text{Absolute unc. } d_t}{d_t} \cdot 100 = \text{Percentage unc. } d_t$$

$$\therefore \text{Percentage unc. } d_t = \frac{0.00067}{1.773} \cdot 100 = 0.0038\%$$

4.2.5 Sensitivity analysis conclusion

The total percentage uncertainty in d_t is:

$$\% \text{ unc } d_t = \% \text{ unc } e + \% \text{ unc angle} + \% \text{ unc } l + \% \text{ unc } h_2$$

$$\therefore \% \text{ unc } d_t = 3.52\% + 0.0012\% + 0.0157\% + 0.0038\% \Rightarrow \% \text{ unc } d_t = 3.57\%$$

$$d_t = 1.773m \pm 3.57\%$$

$$d_t = (1.773 \pm 0.063)m$$

5 Method of experiment:

5.1 Photo of target sheets

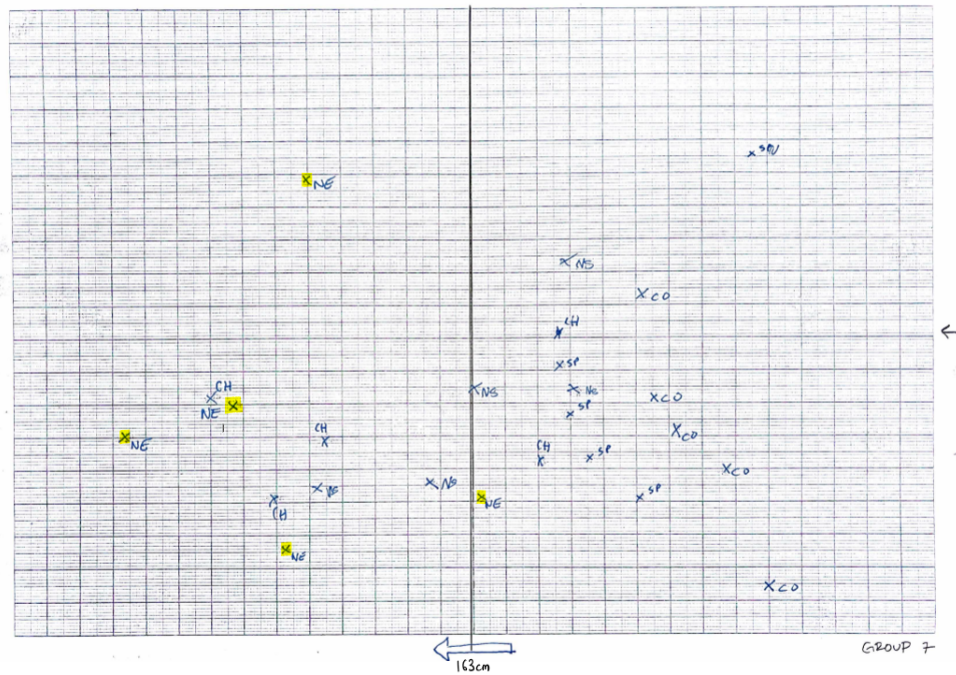


Figure 9: Target sheet (Where the ball landed on the second bounce)

As seen in Figure 7, the target sheet was placed with the middle line at 163cm from the end of the pipe. My data points are highlighted in yellow.

The experimental values inferred from the sheet are laid out in the table below, all values are in cm:

Name	1st	2nd	3rd	4th	5th	Mean	Error	Range
NE (163 cm)	168.1	170.3	173.7	168.7	162.8	168.7	+5.7	10.9
CO (173 cm)	164.0	165.3	166.8	167.8	167.5	166.3	-6.7	3.8
CH (169 cm)	173.5	175.1	176.3	166.9	166.4	171.6	+2.6	9.9
NS (170 cm)	169.9	171.3	174.8	166.9	167.2	170.0	+0.02	7.9
SP (175 cm)	172.3	172.1	171.5	167.9	166.5	170.1	-4.9	5.8

Table 2: Group results from the experiment. Each person's distances are measured from their own predicted value (shown in brackets).

As seen in the table above, NS was the closest to his experimental value, where as CO had the smallest range in her values.

- CO therefore released the ball most consistently, keeping it centred and at rest when released.

- NS was the closest, which either means he released the ball very centred and at rest, or he released it with some initial velocity, which accounted for the assumptions we made.

- One way of ensuring everyone releases the ball correctly would be to install an electromagnet at the top, to remove all human error.

- As CO and I were the last to conduct the experiment, the error causing our experimental values to be below our theoretical values likely came from shifts in the equipment from the original setup, where the angle/height changed over time.

- Finally, the general conclusion in the group was that our theoretical prediction was larger than the experimental value, so I will need to improve my calculations by reducing the assumptions made.

My experimental value of d_t was 168.7 cm, which is 8.6 cm lower than my calculated value.

Comparing this to my uncertainty:

$$\text{Lower bound of } d_t = 1.773 - 0.063 = 1.71\text{m}$$

$$\therefore 1.687\text{m} < 1.71\text{m}$$

As shown above in the calculations, my experimental value lies outside the uncertainty in my theoretical value. This is due to a number of assumptions, and human errors in the experimental method.

5.2 Experimental method

To keep the results similar, everyone in the group largely completed the experiment the same. The discrepancy between members will have caused the differences in accuracy.

Below I have included some photos which were taken during the setup and during the experiment:

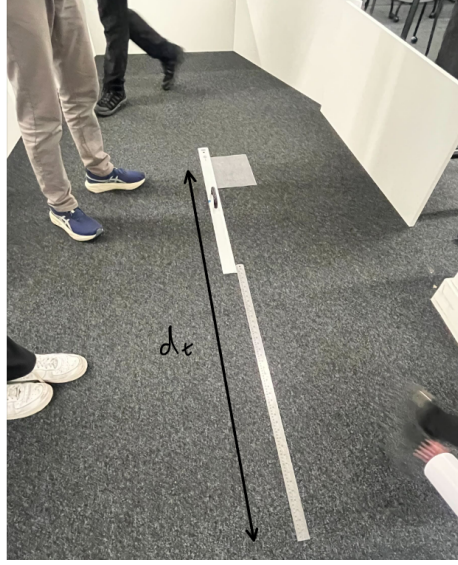


Figure 10: Setting up the target sheet at the correct distance

As seen in Figure 9 below, we ensured the angle was setup to a suitable degree of accuracy, as well as measuring the height of h_2 .

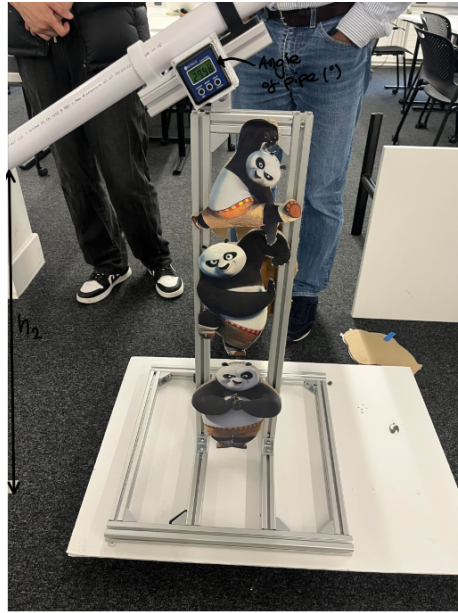


Figure 11: Setting up the angle and height of PVC pipe

5.3 Assumptions

5.3.1 Energy lost in the PVC pipe

These next calculations, regarding sound energy, were made with the help of **ChatGPT** (OpenAI Version 5.1 (November 2025)), with the prompt: Can we estimate the energy

lost due to sound from this video clip, where the phone can be assumed to be held 5 cm away?

An assumption we made was that no energy was lost during the rolling in the tube, where there would be no sound or heat, I analysed a video and produced the graph below:

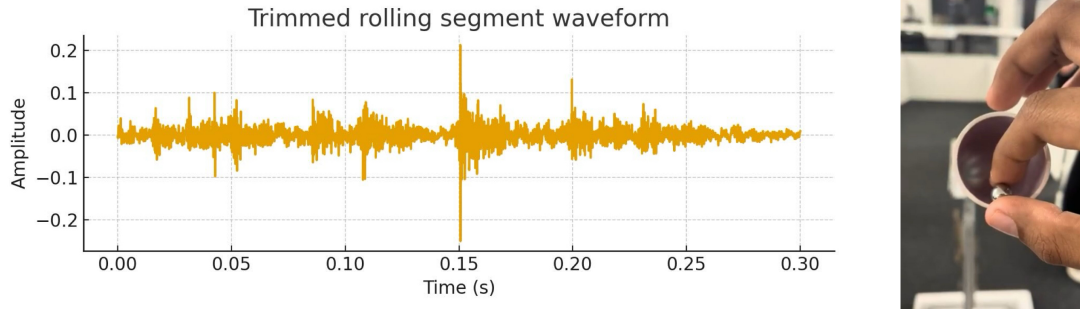


Figure 12: Analysing the sound produced by the ball in the pipe

Making a few assumptions about the video we can calculate a rough energy loss due to sound. Assuming that sound radiates in a sphere, and that the phone is held 5 centimetres away in the video, we can calculate the energy lost:

$$I = \frac{p_{rms}^2}{\rho c}$$

Where I is intensity, ρ is air density, p_{rms} is the root mean squared sound pressure and c is the speed of sound.

Substituting in the approximate values:

$$I = \frac{0.2^2}{1.2 \cdot 343} \approx 9.7 \times 10^{-5} \text{ Wm}^{-2}$$

Using our assumptions:

$$P = I \cdot A$$

$$P = 9.7 \times 10^{-5} \cdot 4\pi(0.05)^2 \approx 2.8 \times 10^{-6} \text{ W}$$

Since $T = 0.9$ seconds:

$$E = P \cdot T$$

$$E \approx 2.8 \times 10^{-6} \cdot 0.9 \approx 2.5 \times 10^{-6} \text{ J}$$

This value compared to the total energy in the initial roll phase is largely negligible, but will still cause the experimental value of d_t to be lower than the theoretical value when ignored.

5.3.2 Initial Kinetic Energy

We also assumed that the ball would be released from rest, but due to human error, this was most likely not the case. In this section I will account for a small starting velocity and how this will impact the final KE.

If we now change our assumption to a small initial speed v_h , along the pipe, we can set out our new conservation of energy as follows:

$$mgh_1 + \frac{1}{2}mv_h^2 = \frac{1}{2}mv_r^2 + \frac{1}{2}I\omega^2$$

Using the same simplification steps as earlier, and $I = \frac{2}{5}mr^2$, $\omega = \frac{v_r}{r}$:

$$v_r^2 = \frac{10}{7}(gh_1 + \frac{1}{2}v_h^2)$$

$$\therefore v_r = \sqrt{\frac{10}{7}(gh_1 + \frac{1}{2}v_h^2)}$$

If we used an ideal case of $v_h = 0$, which we did in Section 1:

$$v_r = \sqrt{\frac{10}{7}gh_1} = 3.242 \text{ m s}^{-1}$$

For this analysis, we will simulate a realistic human error, assuming there is an initial speed of $v_h = 0.5 \text{ m s}^{-1}$, which substituting in gives:

$$v_r = \sqrt{\frac{10}{7}((9.81 \cdot 0.75) + \frac{1}{2}(0.5)^2)} \approx 3.27 \text{ m s}^{-1}$$

This increase in initial velocity causes the final exit speed to go from 3.242 m s^{-1} to 3.27 m s^{-1} , increasing by roughly 0.85%.

If the same steps taken in Section 1 to calculate d_t are taken with this new v_r , we can calculate a new central value of d_t :

$$d_t \approx 1.79 \text{ m}$$

$$\therefore 1.79 - 1.773 = 0.014 \text{ m}$$

This has therefore produced an increase of roughly 0.8% to the total value of d_t , simply because of human error.

Additionally, if v_h was increased to 1 m s^{-1} , the total distance increase would jump up to 3%, roughly equal to a total d_t of 1.83 m.

This shows how much of an impact the human error in the ball's release has on the final distance, explaining the range of values my group had in their experimental values.

5.3.3 Friction during the bounce

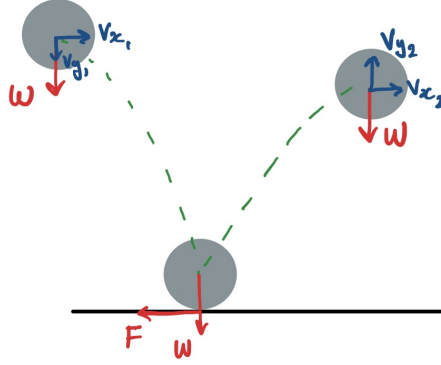


Figure 13: The free body diagram of the ball during its collision with the ground.

We assumed that the collision between the steel ball and the carpet was frictionless, therefore not causing the ball to spin. In this section I will estimate the impact of spin on the distance between bounce one and two (d_3).

$$\omega_0 = \frac{v_r}{r} = \frac{3.242}{0.005} \approx 6.48 \times 10^2 \text{ rad/s}$$

We can assume the angular velocity is constant, as there is no air resistance, allowing us to calculate the spin:

$$u_x 2.808 \text{ m s}^{-1}$$

$$v_{\text{spin}} = \omega_0 \cdot r = 3.242 \text{ m s}^{-1}$$

Since v_{spin} is greater than u_x , friction acts forwards, and therefore is propelled, transferring its rotational kinetic energy into translational kinetic energy.

Estimating $\mu \approx 0.3 - 0.5$ for the contact between carpet and steel, as well as assuming there is no slipping.

For these calculations, we will use our value of u_x from earlier, $u_x = 2.808 \text{ m s}^{-1}$, as well as letting ω_0 be the initial angular velocity, and v_x , ω_1 be the final horizontal speed and spin respectively.

Using momentum, the principle of impulse, and inertia, we can calculate the final horizontal speed:

$$I_{\text{impulse}} = m(v_x - u_x) \quad \& \quad -I_{\text{impulse}} \cdot r = \text{Inertia} \cdot (\omega_1 - \omega_0)$$

where:

$$v_x = \omega_f \cdot r$$

Using the formula for inertia that we have used multiple times, $\text{Inertia} = \frac{2}{5}mr^2$, we can solve:

$$v_x = \frac{5(u_x) + 2(v_r)}{7}$$

$$v_x = \frac{5(2.808) + 2(3.242)}{7} \approx 2.93 \text{ m s}^{-1}$$

This new final value for horizontal velocity means that it has increased:

$$2.808 \text{ m s}^{-1} \Rightarrow 2.93 \text{ m s}^{-1}$$

This increase is therefore 4.4%.

Since the time spent between bounces is the same, as vertical velocity is unchanged, we can calculate a new d_3 , and therefore d_t :

$$d_3 = d_{3\text{original}} \cdot \frac{v_x}{u_x} = 1.083 \cdot \frac{2.93}{2.81} \approx 1.13 \text{ m}$$

Therefore summing d_t :

$$d_t = d_2 + d_3 \approx 0.69 + 1.13 \approx 1.82 \text{ m}$$

This final value compared to my original theoretical prediction:

$$1.82 - 1.773 \approx 0.048 \text{ m}$$

Which is the same as an increase of 2.7%.

This is an upper bound, as a realistic scenario probably lies between both predictions. Therefore, we will instead increase d_t by 0.025m.

5.3.4 Revised additions to sensitivity analysis

Finally, we can combine the three new calculations to get a more accurate value for d_t .

1. Energy lost to sound in the pipe.

$$E_{\text{sound}} \approx 2.5 \times 10^{-6} \text{ J}$$

$$GPE = mgh_1 = 0.00419 \cdot 9.81 \cdot 0.75 \approx 3 \times 10^{-2} \text{ J}$$

$$\therefore \text{Energy lost} = \frac{E_{\text{sound}}}{GPE} \approx 8 \times 10^{-5} = 0.008\%$$

This would change d_t by 0.0001m, which is negligible compared to other results.

2. The addition of initial kinetic energy.

Using the value calculated with $v_h = 0.5 \text{ m s}^{-1}$:

$$\Delta d_t = +0.014 \text{ m}$$

3. The impact of friction during the bounce.

Using the realistic estimate for human error:

$$\Delta d_t = +0.025 \text{ m}$$

Therefore we can calculate the final value of d_t :

$$d_t \approx 1.773 + 0.014 + 0.025 \approx 1.81 \text{ m}$$

This is larger than my original theoretical value of 1.773m, and even larger than my experimental result of 1.687m.

This suggests that the other assumptions such as small impacts between the ball and the PVC pipe and rolling resistance had a big impact on the final experimental value of d_t .

5.3.5 Further assumptions in calculations and experiment

Experimental assumptions:

- **The angle of the pipe remains constant throughout.** This assumption is to do with the fact the pipe is assumed to have not moved throughout the experiment, and realistically it did slightly, although most likely negligible, it will have decreased the value of d_t .

Other assumptions in the calculation:

- **The ball moved side to side in the PVC pipe.** When analysing the video, during the sound analysis, the ball can clearly be seen not moving in a straight line, which will cause the exit velocity out of the pipe to be lower, and therefore d_t to be lower.

- **The ball is assumed to roll down the pipe, without collisions.** This means no free-fall or slipping occurred. The slipping or free-fall would cause collisions with the wall, and the slipping would mean additional friction would occur, reducing the exit velocity of the ball, and therefore decreasing d_t .

- **There is assumed to be no air resistance acting on the ball.** Although the ball is small in diameter, there will still be an impact on the horizontal velocity of the ball, causing a slight constant negative acceleration, decreasing the final value of d_t , however small.

- **It is assumed that the angle of the pipe and the height of the PVC pipe are independent.** I have assumed this in my original sensitivity analysis, therefore rather than affecting the final value of d_t , it affects the uncertainty, meaning the calculated uncertainty is slightly larger than it should be.

- **It is assumed that the carpet is perfectly flat and no friction occurs.** I have assumed the carpet is flat and there is no friction, meaning the bounce has no effect on the horizontal velocity, which in reality is not the case, meaning the final value of d_t is lower than its calculated value.

6 Conclusion

The aim of the project was to accurately predict the total horizontal distance d_t travelled by the steel ball up until the second bounce, after rolling out of a PVC pipe and bouncing once, before comparing this to an experimental value.

My initial calculations using the conservation of energy, kinematics and projectile motion produced:

$$d_t = 1.773\text{m}$$

I conducted a sensitivity analysis, which produced an uncertainty largely from the coefficient of restitution:

$$d_t = (1.773 \pm 0.063)\text{m}$$

The experimental value I got from my 5 trials was:

$$d_t^{\text{experimental}} = 1.687\text{m}$$

This value lay outside of my uncertainty range. This discrepancy made me delve into the reasons this occurred, largely looking at a more realistic model by analysing three additional effects: energy lost to sound in the pipe, a small initial kinetic energy due to human error, and friction in the first bounce allowing rotational KE to turn into translational KE. The largest being the last, with sound energy being negligible. These calculations allowed me to calculate a new theoretical value:

$$d_t^{\text{corrected}} \approx 1.81\text{m}$$

This was interesting, as it was further away from my original prediction and experimental value, indicating that, in reality, a number of resistive forces and effects, particularly in the tube, shortened the experimental value more than I had assumed.

Overall, the calculations and assumptions show that the accuracy throughout is heavily affected by small looking assumptions, such as the initial KE at the start, a human error which is easy to overlook, or the small assumptions about friction and air resistance which can easily add up, causing my final discrepancy.